

**FAR
BEYOND**

MAT122

Integration by Substitution



Stony Brook University

Substitution Rule - Intro

Use **Substitution Rule** when there are 2 functions within the integration
AND one is the derivative of the other.

ex. Find $\int 2x\sqrt{1+x^2} dx$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + C$$

where $n \neq -1$

$$= \frac{2(1+x^2)^{\frac{3}{2}}}{3} + C$$

Substitution Rule

ex. Evaluate $\int (x^3 + 1)^5 3x^2 dx$

$$= \int \textcolor{red}{u}^5 du$$

$$= \boxed{\frac{(\textcolor{red}{x}^3 + 1)^6}{6} + C}$$

ex. Evaluate $\int (x^3 + 1)^5 \textcolor{red}{x}^2 dx$

$$= \boxed{\frac{(\textcolor{red}{x}^3 + 1)^6}{\textcolor{blue}{18}} + C}$$

Substitution Rule – Example #2

ex. Evaluate $\int \frac{x}{\sqrt{1-4x^2}} dx$

$$\text{let } u = 1 - 4x^2$$

$$= -\frac{1}{4} \sqrt{1-4x^2} + C$$

Substitution Rule – Example #3

ex. Calculate $\int e^{5x} dx$

let $u = 5x$

$$= \frac{1}{5} e^{5x} + C$$

Substitution Rule for Definite Integrals

ex. Evaluate $\int_0^4 \sqrt{2x+1} \, dx$

$$\text{let } u = 2x + 1$$

$$= \frac{26}{3}$$

Substitution Rule for Definite Integrals – Example #2

ex. Evaluate $\int_1^2 \frac{dx}{(3-5x)^2}$

let $u = 3 - 5x$

$$= \boxed{\frac{1}{14}}$$

Substitution Rule with $\ln x$

ex. Evaluate $\int_1^e \frac{\ln x}{x} dx$

$$\text{let } u = \ln x$$

$$= \boxed{\frac{1}{2}}$$

ex. Evaluate $\int \frac{9}{8+9x} dx$

$$\text{let } u = 8 + 9x$$

$$= \boxed{\ln|8+9x| + C}$$

Review: Integration with u-Substitution

ex: $\int x \sqrt[3]{1+x^2} dx$

$$= \frac{3}{8} \cdot (1+x^2)^{4/3} + C$$